Study of Turbulent Natural Convection Flow in Rectangular Enclosure

A. I. Fomichev, C. D. Curcija and M.S. Bhandari

Department of Mechanical and Industrial Engineering, University of Massachusetts,
160 Governors Dr., Amherst, MA 01003-9265, USA

Two low-Reynolds-number k-ε turbulence models have been used to predict turbulent natural convection within a differentially heated enclosure for the purpose of assessing the ability of using these models and CFD realized them for simulation of heat-mass transfer in laboratory thermal testing chambers. In both models, the model coefficients vary with the value of the local turbulence Reynolds number. The difference between them is in source term in dissipation equation. The numerical results are compared to published experimental data. Both models predict measured velocity profiles and local Nusselt numbers adequately. The average Nusselt numbers are predicted more accurately for high Rayleigh numbers by the model with added source term than the model without one.

NOMENCLATURE

A	aspect ratio (= H/W)	T	temperature
$C\mu$, C_1, C_2, C_3	constants	T_c	cold-wall temperature
c_l	slope of turbulent length scale in the near-wall region (=2.5) [Eq. (11)]	T_h	hot-wall temperature
f_μ, f_1, f_2	damping functions [Eq. (7), (12) and (13)]	u	horizontal velocity
g	gravity	v	vertical velocity
H	enclosure height	V_{ref}	reference velocity (= $[g\beta(Th\text{-}Tc)H]^{1/2}$)
k	turbulent kinetic energy	W	enclosure width
k_g	gas conductivity	x	horizontal coordinate
LRN	low-Reynolds-number	y	vertical coordinate
n	coordinate direction normal to the nearest wall	α	thermal diffusivity
Nu	local Nusselt number	β	thermal expansion coefficient
p	pseudo-pressure	ε	dissipation of turbulent kinetic energy
P_k	turbulent stress production [Eq. (8)]	ν	molecular kinematics viscosity
q	local heat flux	ν_t	turbulent viscosity
Pr	Prandtl number (= ν/α)	ρ	density
Ra	Rayleigh number based on enclosure height (= $[g\beta(T_h\text{-}t_c)h^3/\nu^2]Pr$)	σ_k, σ_ε	turbulent Schmidt numbers
Re_t	turbulence Reynolds number (= $k^2/\nu\varepsilon$)		

INTRODUCTION

The main aim of this study to assess the ability of a low-Reynolds-number (LRN) k-ε turbulence model to predict heat transfer and fluid flow within a two-dimensional enclosure whose vertical walls are maintained isothermally at different temperatures and horizontal walls are adiabatic. Numerical calculations have been performed for air filled rectangle cavity and are compared with numerical prediction of Heindel et al. [1] and to available experimental measurements taken by Cheesewright et al. [2] and Tian and Karyiannis [3]. Direct comparisons are made between two different LRN models.

Some parts of this work can also be useful as example of checking procedure needed for CFD code validation.

MODEL EQUATIONS

We consider a two-dimensional rectangular cavity with a hot left wall and a cold right wall. The horizontal walls are adiabatic. The height of the cavity is *H* and width is *W*. The flow in the cavity is described by the Reynolds equations applying the Boussinesq approximation:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

X and Y momentum equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\nu+\nu_t)(2\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\nu+\nu_t)(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}) \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+g\beta(T-Tm)+\frac{\partial}{\partial y}(\nu+\nu_t)(2\frac{\partial v}{\partial y})+\frac{\partial}{\partial x}(\nu+\nu_t)(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}) \qquad (3)$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial x}(\frac{\nu}{\mathrm{Pr}}+\frac{\nu_t}{\sigma_T})\frac{\partial T}{\partial x}+\frac{\partial}{\partial y}(\frac{\nu}{\mathrm{Pr}}+\frac{\nu_t}{\sigma_T})\frac{\partial T}{\partial y} \qquad (4)$$

Turbulent kinetic energy transport equation:

$$\frac{\partial k}{\partial t}+u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\frac{\partial}{\partial x}(\nu+\frac{\nu_t}{\sigma_k})\frac{\partial k}{\partial x}+\frac{\partial}{\partial y}(\nu+\frac{\nu_t}{\sigma_k})\frac{\partial k}{\partial y}+P_k+G_k-\varepsilon \qquad (5)$$

Dissipation of turbulent kinetic energy transport equation:

$$\frac{\partial \varepsilon}{\partial t}+u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial x}(\nu+\frac{\nu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x}+\frac{\partial}{\partial y}(\nu+\frac{\nu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial y}+C_1 f_1\frac{\varepsilon}{k}P_k+C_2 f_2\frac{\varepsilon^2}{k}+C_3\frac{\varepsilon}{k}G_k+E_\varepsilon+S_\varepsilon \qquad (6)$$

The eddy viscosity is obtained from *k* and ε by the Prandtl-Kolmogorov relation,

$$\nu_t = C_\mu f_\mu \frac{k^2}{\varepsilon} \tag{7}$$

The stress production term, P_k, is modeled by expression

$$P_k = \nu_t \left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}\right)^2 \right] \tag{8}$$

The buoyancy term, G_k, is modeled by

$$G_k = -g\beta \frac{\nu_t}{\sigma_t}\frac{\partial T}{\partial y} \tag{9}$$

The source term E_ε in the ε equation is defined in LRN k-ε model of Jones and Launder [4] (1972) as

$$E_\varepsilon = 2\nu\nu_t \left[\left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \left(\frac{\partial^2 v}{\partial y^2}\right)^2 \right] \tag{10}$$

The source term S_ε in the ε equation proposed by Yap [5] and then used by Ince and Launder in [6] is defined as

$$S_\varepsilon = 0.83\left(\frac{k^{3/2}}{\varepsilon c_l y} - 1\right)\left(\frac{k^{3/2}}{\varepsilon c_l y}\right)^2 \frac{\varepsilon^2}{k} \tag{11}$$

where y is distance from the wall and c_l (=2.5) is a slope of turbulent length scale $k^{3/2}/\varepsilon$ in the near-wall region of a constant-stress shear flow.

From these model equations we formed two LRN k-ε turbulent models the first of them where terms E_ε and S_ε are omitted is identical to LRN k-ε model with variable coefficients described in details in [1] as VC LRN k-ε model. Our second LRN k-ε model is the same model in which we included terms E_ε and S_ε. To differentiate between the different LRN k-ε models in this study we will name the first model as LRN k-ε model N1 and the second – as LRN k-ε model N2.

<u>Model Constants</u>

In this study for both LRN k-ε models we used the following constants and relationships:
$C_\mu = 0.09$; $\sigma_t = 1.0$; $\sigma_k = 1.0$; $\sigma_\varepsilon = 1.3$; $C_1 = 1.44$; $C_2 = 1.92$; $C_3 = \tanh|v/u|$; $f_1 = 1.0$;

$$f_\mu = \exp[-3.4/(1 + Re_t/50)^2]; \tag{12}$$

$$f_2 = 1.0 - 0.3\exp(-Re_t^2), \tag{13}$$

where $Re_t = k^2/(\nu\varepsilon)$ is the turbulence Reynolds number.

<u>Boundary Conditions</u>

In the both LRN k-ε models, the boundary conditions for k and ε are set at the wall, where k is set to zero and value for ε is obtained according [7] from

$$\varepsilon\,|_w = 2\nu\left(\frac{\partial\sqrt{k}}{\partial n}\right)^2 \qquad (14)$$

where $\varepsilon\,|_w$ is the value of ε at the wall and *n* is the coordinate direction normal to the wall.

NUMERICAL METHOD

A computer code named FLU2TURB has been developed to solve the two-dimensional steady turbulent problem. The numerical discrete method is based on the upwind and fully implicit transient differencing control volume scheme used respectively for the convective, diffusive and time-dependent terms in the governing equations where the velocity control volumes are staggered with respect to the main control volumes. The resulting algebraic equations are solved iteratively using a line-by-line TDMA solution procedure and the SIMPLE algorithm formulated by Patankar [8]. Steady-state solutions are obtained using under-relaxation techniques.

SOLUTION PROCEDURE AND NUMERICAL ACCURACY

We have used relaxation factors of 0.5 for velocities and turbulent values (*k* and ε) and 0.8 for ν_t, pressure and pressure correction. After 2500 iterations, the relaxation factors for turbulent values and velocities were reduced to 0.2. Total number of iterations depended from Rayleigh number of the problem and changed in the range from 3000 to 9000 iterations until convergence criterion was satisfied.
Solutions were considered to be converged if the next criteria were satisfied: the residual mass source was less than 10^{-7}, and changes in computed values of velocity, kinetic energy k, pressure and difference between input and output heat over 20-iteration cycle were less than 0.01%, 0.1%, 0.1% and 0.5% respectively when compared to the maximum value in the computational domain. The solution procedure has been validated against the benchmark results of Davis [9] for laminar natural convection in square cavity with very good agreement.

We used strongly no uniform 28x55 and 30x65 grids modeling Cheesewright et al. [2] conditions. The grid distribution provided at least five nodes in inner layer that we considered sufficient to capture the physics in the wall region. For these grids we obtained a 2% difference in average Nusselt number (203 and 199 respectively) using LRN k-ε model N1 and less than 2% difference in the maximum velocities. Velocities profiles, velocities fields and various turbulent-related values were the same for both grids but location of the transition from laminar to turbulent natural convection was changed that confirms results of another numerical studies [1, 10] about non-uniqueness of the transition location. In experiment works transition is known to occur over a range of vertical distances, depending on the physical conditions of apparatus.
In any case it is necessary to verify the numerical accuracy of the predicted results refining the grid. Usually a several quantities need to compare: averaged wall heat transfer ($\overline{Nu}$), the maximum vertical velocity and turbulent viscosity at half the cavity height. For expanding study the maximum horizontal velocity at half the cavity width and the vertical gradient of the thermal stratification at the cavity center can be also taken. As example in figure 1 we show relationship between average Nusselt number and various grids for cavity with aspect ratio A = 2.6 and Rayleigh number Ra = $5.0\text{x}10^9$. The results were obtained using LRN k-ε model N1.

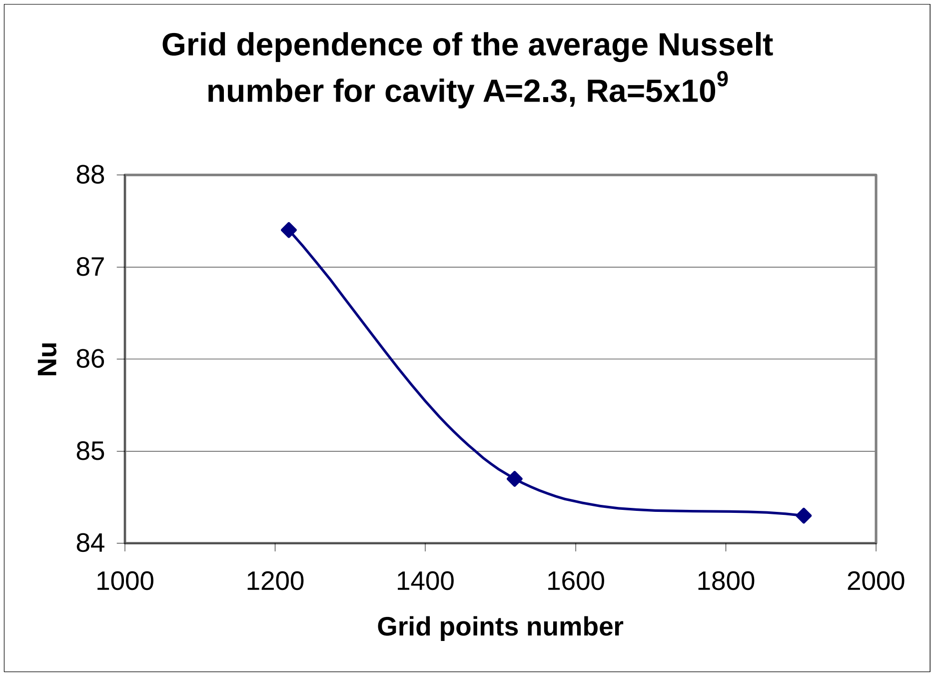

Figure 1. Average Nusselt number asymptotic behavior.

RESULTS AND DISCUSSION

The model equations, the empirical relations and boundary conditions (1) – (14) were used to predict the fluid flow and heat transfer within a differentially heated enclosure (Figure 2). The vertical walls were maintained isothermally at Th and Tc and the horizontal walls were adiabatic. The conditions matched the experimental values of Cheesewright et al. [2] by setting Ra = $5x10^{10}$, Pr = 0.71 and aspect ratio A = 5. The Rayleigh number was scaled using the enclosure height H as length scale. The reason for this approach is that the core of the enclosure is stratified, with horizontal isotherms and streamlines. Therefore the core structure is essentially one- dimensional (depending only on Y coordinate) than the width of the enclosure does not appear explicitly in the scaling.

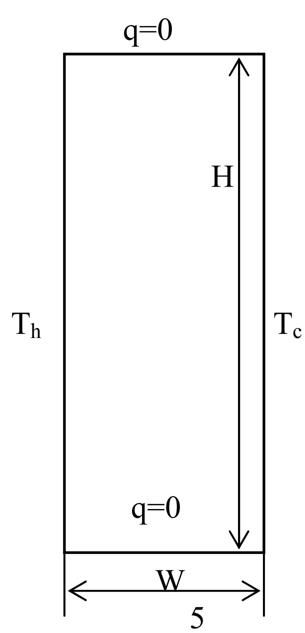

Figure 2. Geometry and boundary conditions of the modeled enclosure.

The first results obtained by using LRN k-ε model N2 with S_ε term given by (11) showed that this model gives strongly asymmetrical distorted picture of distribution turbulence quantities in enclosure, although average Nusselt number is good (for example Nu = 173 for Ra = $5\text{x}10^{10}$ and A=5). That is why we decided to replace original term (11) in LRN k-ε model N2 by the following

$$S_\varepsilon = 0.83\left(\frac{k^{3/2}}{\varepsilon c_l y} - 1\right)\left(\frac{k^{3/2}}{\varepsilon c_l y}\right)\frac{\varepsilon^2}{k} \tag{15}$$

The ratio of the eddy viscosity to the molecular kinematics viscosity (ν_t/ν) is a relative measure of the diffusive potential of the turbulence in enclosure. For Cheesewright et al. [2] case the maximum values predicted by the LRN k-ε model N1 and LRN k-ε model N2 are 41.5 and 57.8 respectively. For VC LRN k-ε model in [1] maximum value 44.2 was obtained. The LRN k-ε model N1 predicts a smaller turbulent region and vertical location of the large increasing of ν_t/ν is higher than for LRN k-ε model N2. Contours similar to those of Figure 3 were obtained for the turbulence Reynolds number, since Re_t, is proportional to ν_t. The maximum values of Re_t predicted by LRN k-ε models N1 and N2 were 480 and 650 respectively. For VC LRN k-ε model in [1] maximum value Re_t = 500 was obtained. Both models predicted values normalized turbulent kinetic energy $\mathbb{k}$ = k/[gβ(Th-Tc)H] in the range of $2.1\text{x}10^{-3}$ to $2.8\text{x}10^{-3}$.

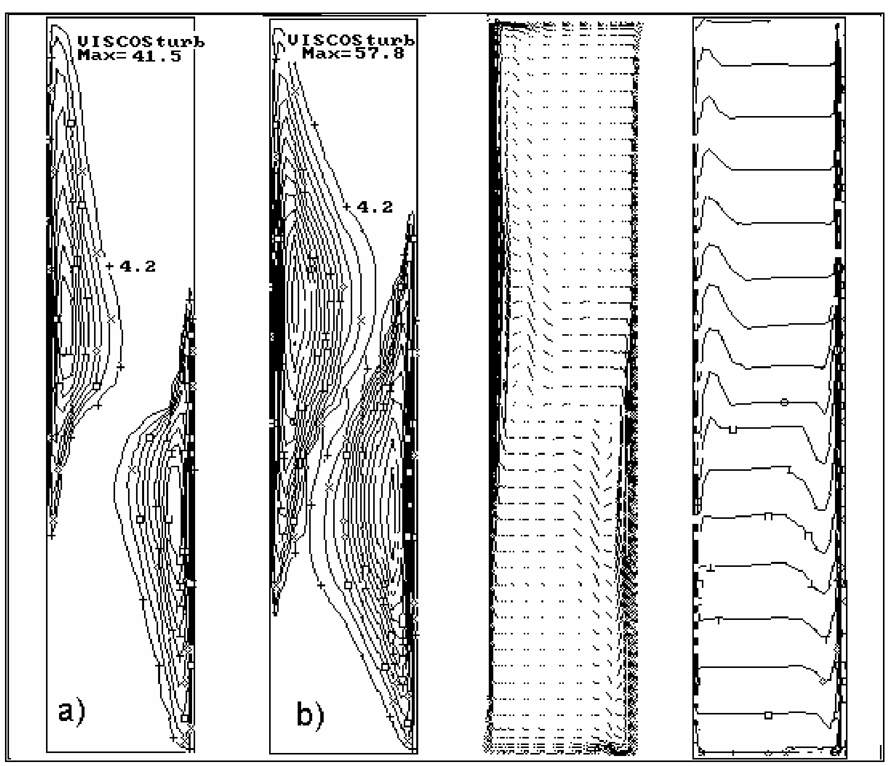

Figure 3. Contours of ν_t/ν (a) for the LRN k-ε model N1 and (b) for the LRN k-ε model N2, velocity and temperature plot for Ra = $5\text{x}10^{10}$ and A = 5.

To assess the accuracy of the turbulence models further, predictions are compared to the experimental results of Cheesewright et al. [2]. Figure 5 displays the predicted and measured relative vertical velocity V/V_{ref} at an enclosure at the location Y = 0.5, where a reference velocity V_{ref} is defined by $V_{ref} = [g\beta(Th-Tc)H]^{1/2}$. The LRN k-ε model N1 follows the measured profile throughout the enclosure width, while the LRN k-ε model N2 predicts a thicker boundary layer. The agreement between both LRN k-ε models and the experimental data is good. Figure 6 displays velocity profiles at the location of transition from laminar to turbulent natural convection. The LRN k-ε model N1 gives a slightly higher velocity in boundary layer along hot wall but both predicted profiles are within the bounds of experimental uncertainty.

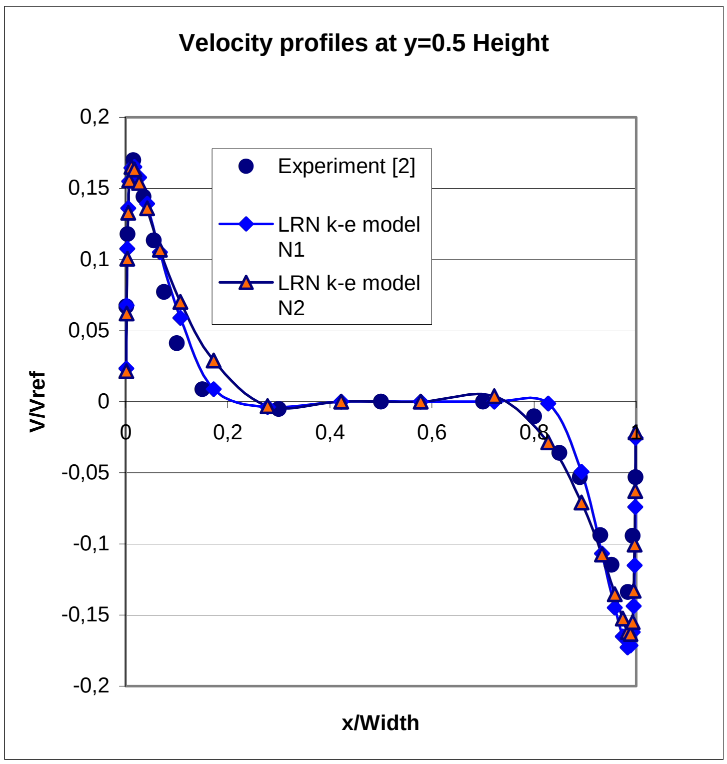

Figure 5. Predicted velocity profiles and experimental data [2] at the enclosure mid-height.

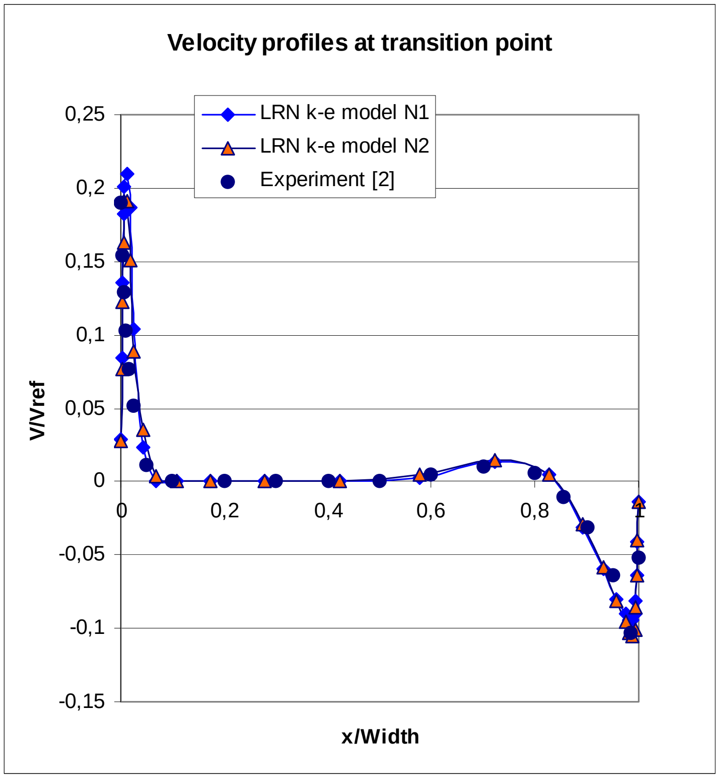

Figure 6. Predicted velocity profiles and experimental data [2] at the location of transition from laminar to turbulent natural convection.

The mean temperature distribution at the enclosure mid-height is frequently used to describe the thermal field in cavity. The predicted temperature profile at the enclosure mid-height by both LRN k-ε models is shown in Figure 7.

In core area of the enclosure, the fluid is stratified. Figure 8 shows that along the mid-width of the enclosure and for most part, the temperature is almost linear distribution. The vertical gradient of the temperature stratification in the enclosure center usually named stratification parameter (S_p) and defined as

$$S_p = [H/(T_h - T_c)/\partial T/\partial y] \qquad (16)$$

is 0.61 for LRN k-ε model N1 and 0.33 for LRN k-ε model N2.

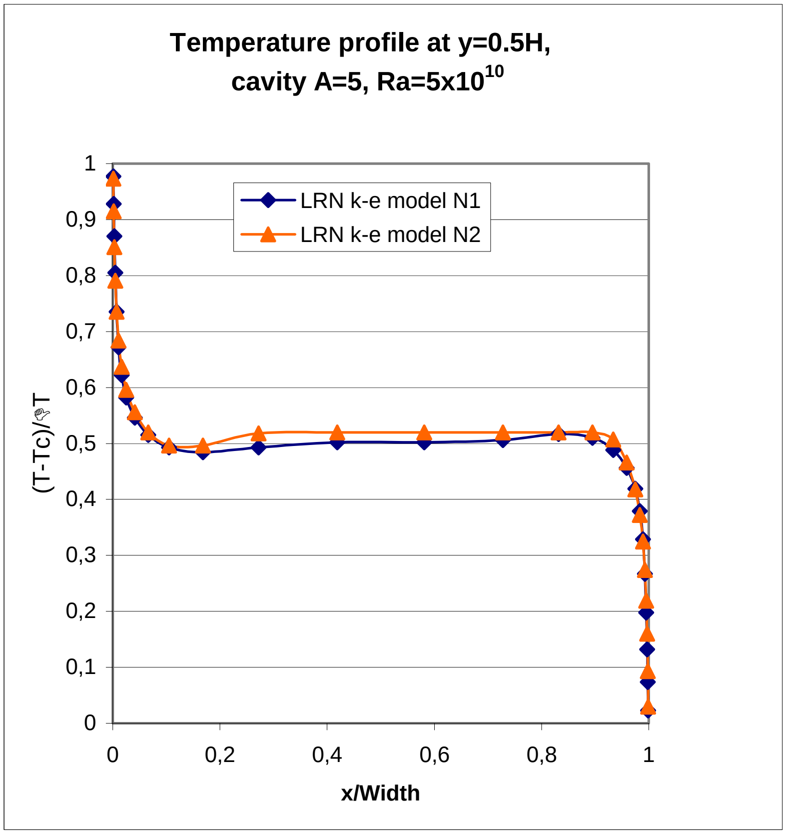

Figure 7. Temperature profile at the enclosure mid-height for Ra = $5x10^{10}$ and A=5.

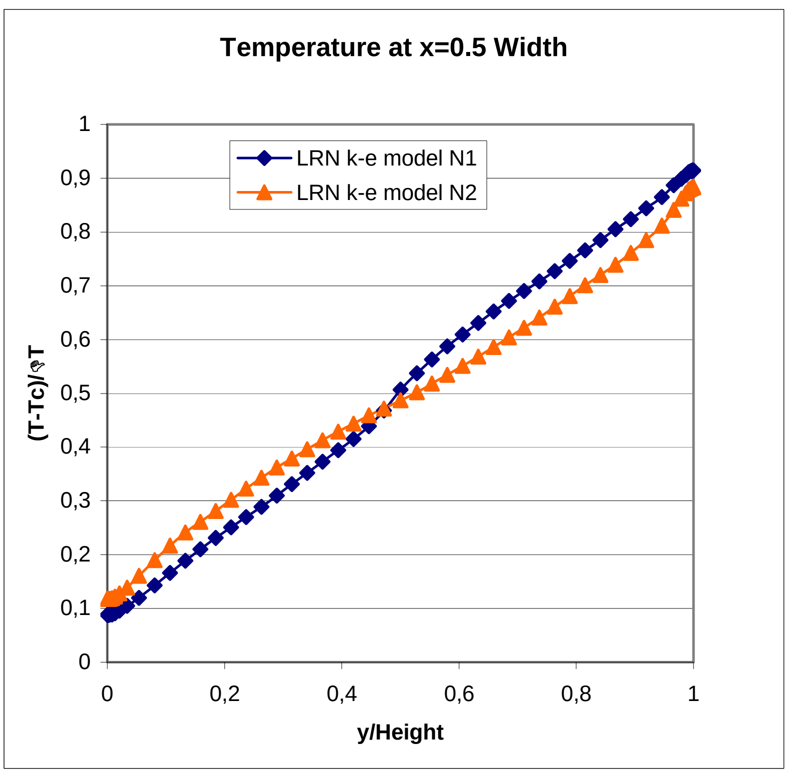

Figure 8. Temperature distribution at the enclosure mid-width for Ra = $5x10^{10}$ and A=5.

A comparison of predicted local Nusselt numbers along the hot wall for Cheesewright et al. [2] conditions is shown in Figure 9. The local Nusselt number is defined by a ratio of local heat flux (q) and reference heat flux

$$Nu = q / [k_g (Th - Tc)/H] \qquad (17)$$

where k_g is gas conductivity for average temperature of cavity. Both models predict almost identical Nusselt numbers at the beginning of the boundary layer but at transition points (Figure 9) model predictions begin to rise and then slowly decline. Transition point corresponds to location where ν_t/ν values begin to increase and may be associated with the differentiation between laminar and turbulent natural convection along the enclosure wall. According to Cheesewright et al. [2] a beginning of transition has been indicated at $y/H \approx 0.22$. The LRN k-ε model N1 predicts location of transition at $y/H = 0.28$ and LRN k-ε model N2 – at $y/H = 0.19$.

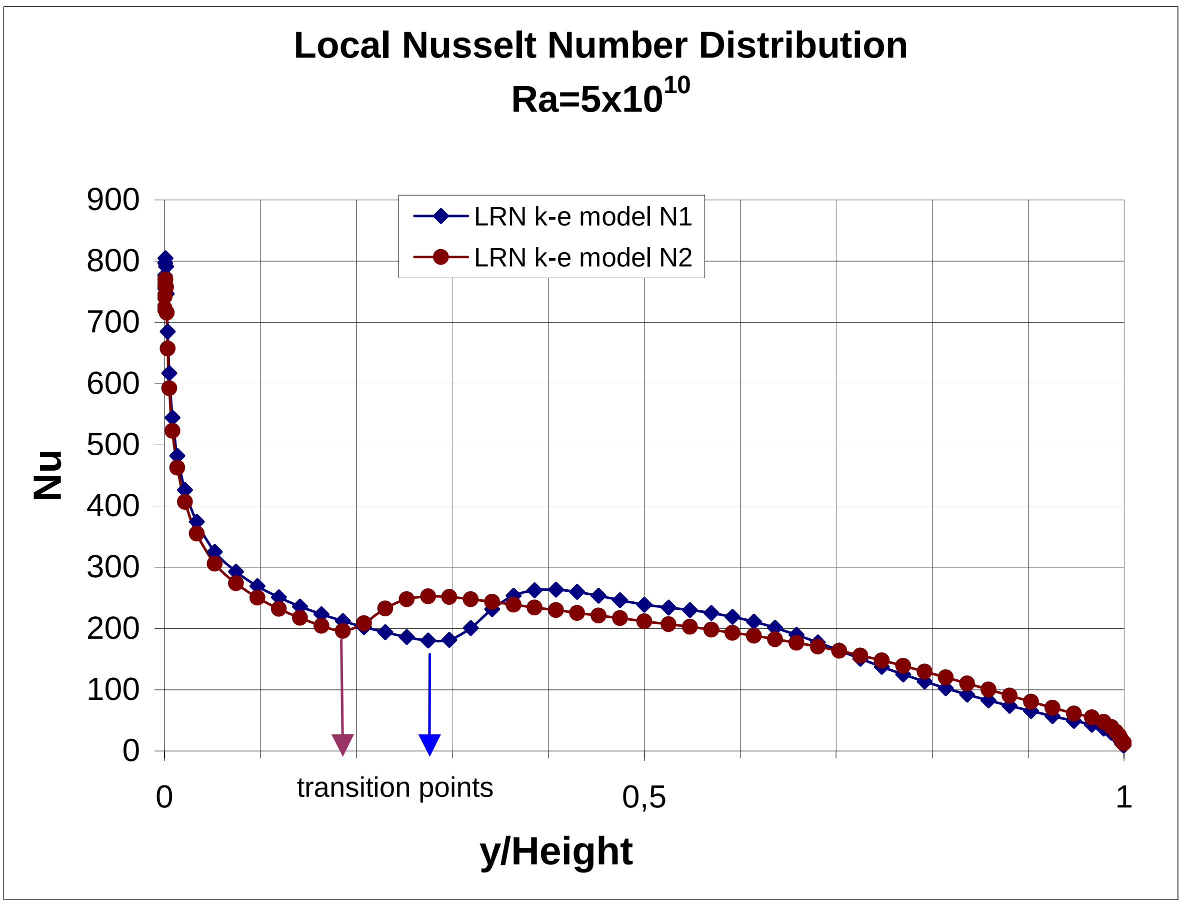

Figure 9. Local Nusselt number distribution along the hot wall for Ra = 5^x10^{10}, A=5.

In Table 1, predicted average Nusselt numbers are compared with values obtained from numerical study [1] and experimental studies [2] and [3]. The excellent conducted experimental study of low- level turbulence natural convection in square cavity [3] can be useful for CFD code validation. In spite of used in

[3] another boundary conditions (LPT: linear temperature distribution or conducting horizontal walls) than in present study we compared predicted by both models average Nusselt numbers because LPT and adiabatic boundary conditions give little change in the value of average Nusselt number. Table 1 shows that both LRN k-ε models predict average Nusselt numbers very closely to those determined from numerical and experimental studies.

Table 1. Comparison of predicted average Nusselt number with numerical and experimental data.

Source	Average Nu number	
	Ra = 5^x10^{10}, A=5, air	Ra = 1.58^x10^9, A=1, air
Experimental work [2]	182*	-
Experimental work [3]	-	64.65*
Numerical study [1]	199.8	-
Current LRN k-ε model N1	199.0	62.2
Current LRN k-ε model N2	195.3	62.1

*Notes: given mean values between hot and cold walls based on the reported values of average Nu number of 154 and 210 for hot and cold walls in [2] and the corresponding values of 64.0 and 65.3 in [3].

To assess Rayleigh-number dependence of average Nusselt number predicted by LRN k-ε models calculations were carried out for natural convection in an enclosure in the range Rayleigh number from $1.1x10^9$ to $2.5x10^{11}$. In Figure10 results give along with the best fit experimental correlation of average Nusselt number for the plate $\overline{Nu}$ =0.047 $Ra^{1/3}$ according to [11]. In the most part of the studied range the values of average Nusselt number predicted by both models are close to each other and over prediction not more than 17%, although for Ra = $2.5x10^{11}$ LRN k-ε model N1 over predicts average Nusselt number twice more than LRN k-ε model N2.

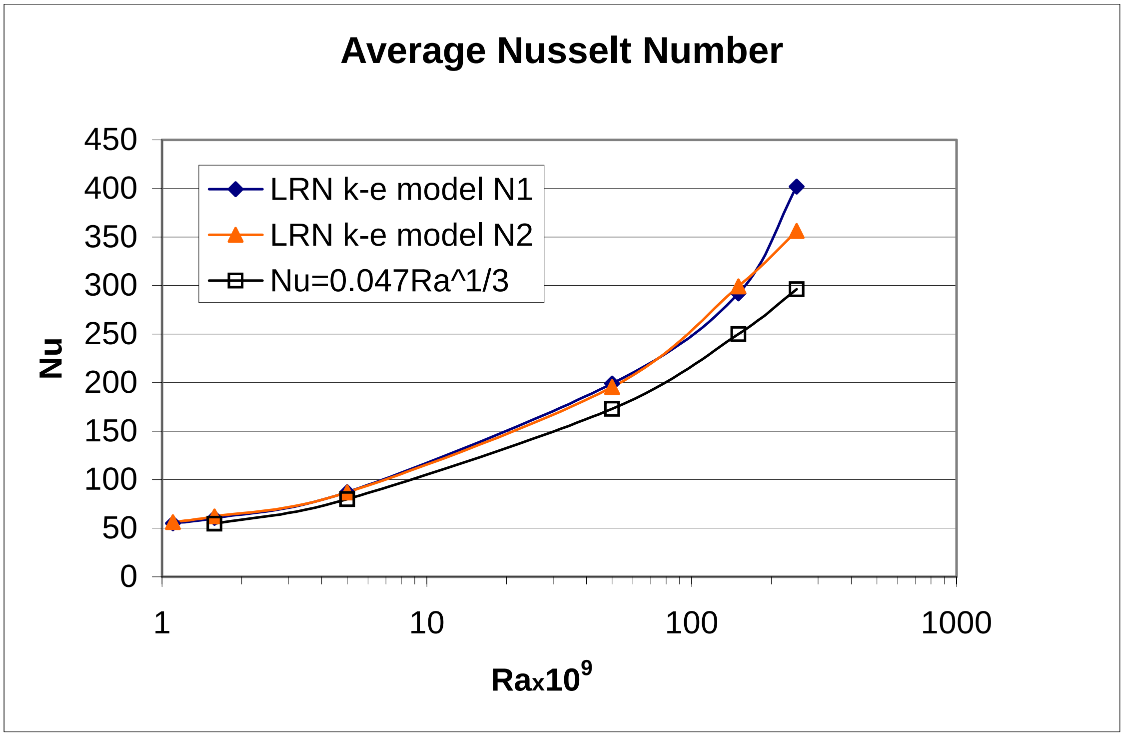

Figure 10. The comparison of Rayleigh-number dependence of average Nusselt number predicted by LRN k-ε models with experimental correlations.

CONCLUSIONS

Two low-Reynolds-number k-ε turbulence models were used to predict turbulent natural convection within a differentially heated enclosure to assess the ability of using these models for simulation of heat-mass transfer in laboratory thermal testing chambers.

Both models predicted laminar and turbulent flow regimes, but in general, the LRN k-ε model N1 predicted lower turbulence levels and smaller regions of turbulent natural convection. The maximum turbulent kinetic energies predicted by both models were of the same order of magnitude but maximum ν_t/ν values predicted by the LRN k-ε model N2 exceeded those of the model N1 by 38% (Ra = $5x10^{10}$, A=5).

When compared to the experimental results of Cheesewright et al. [2], both models satisfactorily predicted the velocity profiles and maximal velocity values.

The average Nusselt number predicted by both models is very close to existing experimental correlations and available data.

From the foregoing results, it is concluded that the LRN k-ε model N1 does a satisfactory job of predicting natural convection flow and heat transfer within a differentially heated enclosure in the range Ra number until $2.0x10^{11}$. The LRN k-ε model N2 can be recommended to use in the range Ra number over $2.0x10^{11}$.

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